Math 75 Calculus I/Burger | The Chain Rule Project | Worth 30 pts.

Show your work for full credit. | Due: Oct. 2, 2015 in class

You are encouraged to work in groups but must turn in your own write-up.

1. Let $f(x) = (3x^2 + 1)^2$. We are going to find the derivative of $f(x)$ in three ways and then compare the answers.

   (a) Algebraically multiply out the expression for $f(x)$ and then take the derivative.

   (b) View $f(x)$ as a product of two functions, $f(x) = (3x^2 + 1)(3x^2 + 1)$ and use the product rule to find $f'(x)$.

   (c) Apply the chain rule directly to the expression $f(x) = (3x^2 + 1)^2$

   (d) Are your answers in parts a, b, and c the same? Why or why not?

2. Let $f(x) = \sin(2x)$. We are going to find the derivative of $f(x)$ in two ways and then compare answers. You will need the double angle formulas for this problem:

- $\sin 2x = 2 \sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$

(a) Rewrite $\sin(2x)$ using the double-angle formula, then apply the product rule to find $f'(x)$.

(b) Apply the chain rule directly to the expression $f(x) = \sin(2x)$ to find its derivative a second way.

(c) Are your answers in parts a and b the same? Why or why not?

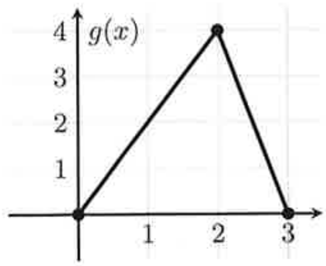
3. Suppose f is differentiable and that $g(x) = (f(\sqrt{x}))^3$.

(a) Calculate $g'(x)$.

(b) If $f(2) = 1$ and $f'(2) = -2$, calculate $g'(4)$.

4. Let $f(x)$ and $g(x)$ be two functions. Values of $f(x)$ and $f'(x)$ are given in the table below and the graph of $g(x)$ is as shown.

x	1	2	3
$f(x)$	3	2	1
$f'(x)$	4	5	6



(a) Let $h(x) = g(f(x))$. Find $h'(3)$.

(b) Let $k(x) = f(g(x))$. Find $k'(1)$.

5. The US population on July 1 of 2010 was 309.33 million. The population was 311.59 million on July 1 of 2011.

(a) Find an exponential model $p(t)$ to fit this data. Let $t = 0$ on July 1, 2010.

(b) Use your model to estimate US population on November 1 of 2013.

(c) Find $p'(3)$. Interpret the meaning of this number, including units.

6. Chains, Inc. is in the business of making and selling chains. Let $c(t)$ be the number of miles of chain produced after t hours of production. Let $p(c)$ be the profit as a function of the number of miles of chain produced, and let $q(t)$ be the profit as a function of the number of hours of production.

(a) Suppose the company can produce three miles of chain per hour, and suppose their profit on the chains is \$4000 per mile of chain. Find each of the following (include units).

$c(t) =$

$c'(t) =$

Meaning of $c'(t)$:

$p(c) =$

$p'(c) =$

Meaning of $p'(c)$:

$q(t) =$

$q'(t) =$

Meaning of $q'(t)$:

How does $q'(t)$ relate to $p'(c)$ and $c'(t)$?

(b) In this part, the production and profit functions are no longer linear. Instead $p(c)$ is modeled by the formula $p(c) = 100 - 100\cos\left(\frac{\pi}{38}c\right)$ (where p is measured in thousands of dollars and c is measured in miles of chain), and $c(t)$ is defined numerically below:

t (in hours)	2	4	6	8	10
c (in miles)	6	14	24	38	52

Estimate $q'(4)$ and $q'(8)$. What conclusions should you draw about production?